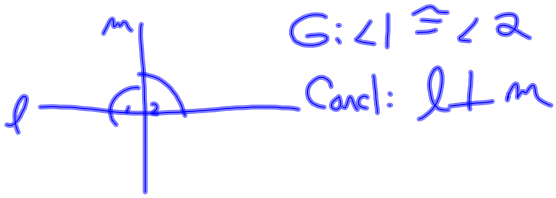
3.6 Prove Theorems about Perpendicular Lines

Notes

Several theorems about perpendicular lines

Theorem 3.8—If 2 lines intersect to form a linear pair of congruent angles, then the lines are perpendicular.

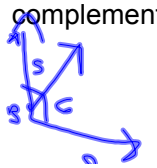


Theorem 3.9—If 2 lines are perpendicular, then they intersect to form four right angles.

(def of ⊥ lines) G: l ⊥ m

Complement Thm

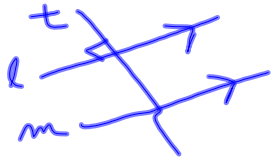
Theorem 3.10—If 2 sides of 2 adjacent acute angles are perpendicular, then the angles are complementary.



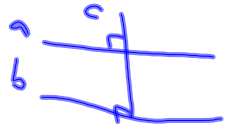
G: $\overline{AB} \perp \overline{BC}$

C: ∠5 & ∠6 are compl.

Theorem 3.11—Perpendicular Transversal Theorem—If a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other.



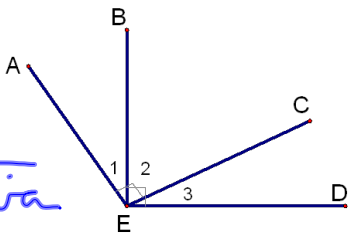
Theorem 3.12—Lines Perpendicular to a Transversal Theorem—In a plane, if two lines are perpendicular to the same line, then they are parallel to each other.



Example:

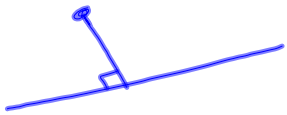
Given: $\overline{AE} \perp \overline{EC}$; $\overline{BE} \perp \overline{ED}$

Prove: ∠1 ≅ ∠3

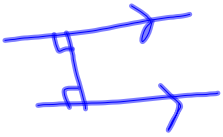


Statements	Reasons
① ~	① Given
② ∠2 & ∠3 are compl ∠1 & ∠2 are compl	② If 2 sides of 2 adj acute ∠s are ⊥, then ∠s compl.
③ ∠1 ≅ ∠3	③ ≅ Compl. thm

Distance from a point to a line—the length of the perpendicular segment from the point to the line.



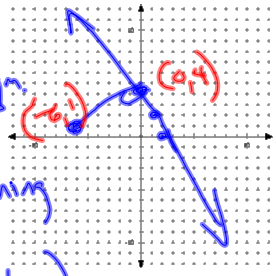
The distance between 2 parallel lines is the length of any perpendicular segment joining the 2 lines.



Examples:

Find the distance between the point and the line listed.

Ex 1: (-6,1) and y = -2x + 4



① Where does ⊥ segm intersect line?

② Eqn of line containing ⊥ segment

③ Solve system (of 2 lines)

④ Distance formula

$m = \frac{1}{2}$

$y = \frac{1}{2}x + b$

$+1 = \frac{1}{2}(-6) + b$

$4 = b$

$\begin{cases} y = \frac{1}{2}x + 4 & (\perp \text{line}) \\ y = -2x + 4 & (0,4) \end{cases}$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

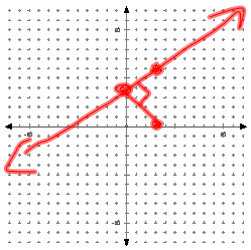
$(-6,1)\ (0,4)$

$d = \sqrt{(-6-0)^2 + (1-4)^2}$

$d \approx 6.71$ units

$d = 3\sqrt{5}$

Ex 2: (3, 0) and $y = \frac{2}{3}x + 4$



Eqn ⊥ segment

$y = -\frac{3}{2}x + b$

$0 = -\frac{3}{2}(3) + b$

$\frac{9}{2} = b$

$\begin{cases} y = -\frac{3}{2}x + \frac{9}{2} \\ y = \frac{2}{3}x + 4 \end{cases}$ → Solve this system

$\left[-\frac{3}{2}x + \frac{9}{2} = \frac{2}{3}x + 4\right] 6$

$-9x + 27 = 4x + 24$

$3 = 13x$

$\frac{3}{13} = x$ $\left(\frac{3}{13}, \frac{54}{13}\right)$

$\frac{54}{13} = y$ $(3, 0)$

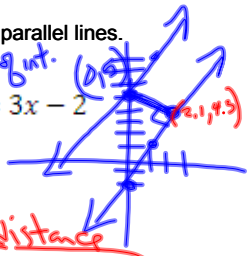
$d = \sqrt{\left(3 - \frac{3}{13}\right)^2 + \left(\frac{54}{13}\right)^2}$

≈ 4.99

Find the distance between the two parallel lines.

Choose the y-int as pt & int.

Ex 3: $y = 3x + 5$ and $y = 3x - 2$



$\begin{cases} y = -\frac{1}{3}x + 5 \\ y = 3x - 2 \end{cases}$

$3x - 2 = -\frac{1}{3}x + 5$

$9x - 6 = -1x + 15$

$10x = 21$

$x = 2.1$

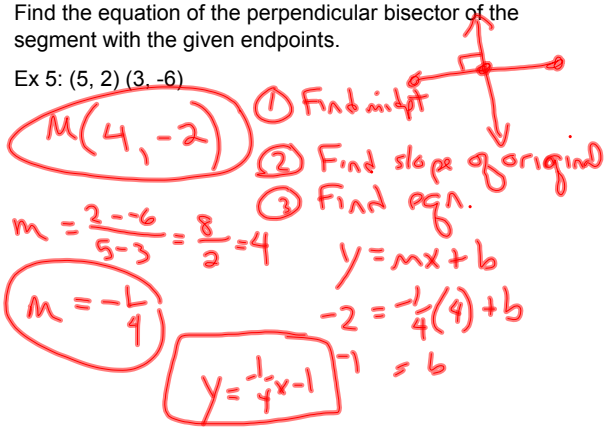
(2.1, 5.3)
(0, 5)

$d \approx 2.21$ units

Ex 4: $y = \frac{-1}{2}x$ and $y = \frac{-1}{2}x + 6$

Find the equation of the perpendicular bisector of the segment with the given endpoints.

Ex 5: (5, 2) (3, -6)



Ex 6: (-3, 7) (8, -5)

HW p194-195 #s 1-7, 13-17 ex 4 ex 6
and find the distance between

$y = -2x + 4$

$y = -2x - 5$

Test Thursday!